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Stirring and mixing: What are the rate-controlling processes?

Chris Garrett

Department of Physics and Astronomy, University of Victoria, Victoria, British Columbia, Canada

Abstract. The parameterization of unresolved processes in oceanographic models is complicated by the interplay of processes on a wide variety of space and time scales. The lack of spectral gaps further complicates the situation, though perhaps not as seriously as might be feared. Understanding the interaction of different processes, and determining which one is critical, or rate-controlling, may be aided by thinking in terms of a triple decomposition into mean, eddies and turbulence. Particular physical processes reviewed include the fate of energy released in baroclinic instability and the ultimate thermohaline frontolytic mechanism on mean isopycnal surfaces. Both of these issues may have implications for diapycnal mixing rates. Another important question reviewed briefly is that of the efficiency of conversion of turbulent kinetic energy to mean potential energy; we do not know what "external" parameters determine it. The kinematic details of overturning are also discussed in terms of the probability distribution of displacements from a stably stratified buoyancy profile.

1. Introduction

The behavior of the ocean is affected by a variety of processes occurring at space and time scales that are too small to be resolved explicitly in models. The influence of these processes then needs to be parameterized in terms of variables that are explicitly included in the models.

Following a "Reynolds decomposition" of variables into a (slowly changing) mean and fluctuations, we therefore seek a representation of the eddy fluxes $\overline{u_i'C'}$ for a scalar C and $\overline{u_i'u_j'}$ for momentum, where the prime superscript denotes the fluctuation. For the scalar C it is usual to assume that the eddy flux is related by a tensor to the local gradient of the mean $\overline{C}$, as would be appropriate in a mixing length theory in which the fluctuations are related to particle displacements that are small compared with the distance over which the mean changes significantly. Thus

$$\overline{u_i'C'} = -T_{ij}\partial\overline{C}/\partial x_j. \qquad (1)$$

The symmetric part K_{ij} of T_{ij} is diagonalizable and is likely to represent down-gradient diffusion parallel to the principal axes of the tensor. The antisymmetric part S_{ij} of T_{ij} has an associated "skew flux" $\mathbf{F}_s$ given by

$$F_{si} = -S_{ij}\partial\overline{C}/\partial x_j = (\mathbf{D} \times \nabla\overline{C})_i \qquad (2)$$

where $\mathbf{D} = -(S_{23}, S_{31}, S_{12})$. This flux is perpendicular to $\nabla\overline{C}$ and may be written as

$$\mathbf{F}_s = -(\nabla \times \mathbf{D})\overline{C} + \nabla \times (\mathbf{D}\overline{C}). \qquad (3)$$

The second term of this is non-divergent and so does not affect the evolution of $\overline{C}$. The first term is advective with a velocity $\mathbf{U}_s$ which may be written

$$U_{si} = \partial S_{ij}/\partial x_j. \qquad (4)$$

This standard formalism (e.g., *Rhines and Holland* 1977; *Moffatt* 1983; *Middleton and Loder* 1989) is purely kinematic. In practice, however, it seems likely that K_{ij} describes large mixing rates along mean isopycnals and a very much smaller diapycnal mixing rate, whereas $\mathbf{U}_s$ is related to the difference between Lagrangian and Eulerian mean flows. Values for K_{ij} and $\mathbf{U}_s$ appropriate for the oceans in their present state could conceivably be obtained from appropriate and sufficient observations. However, extrapolation of limited measurements and the requirements of models that seek to be predictive for different ocean states require that we obtain formulae for K_{ij} and $\mathbf{U}_s$ in terms of resolved variables. This, in turn, requires that we understand the processes responsible for the fluxes.

Many problems immediately arise. One is the validity of a representation of the fluxes in terms of local mean gradients. There are clearly examples in the ocean in which ocean properties are advected by coherent eddies over larger distances than those over which mean gradients are reasonably constant. This is likely to be even more of a problem for momentum, which may be carried long distances by waves. Such situations call for different forms of parameterization but will not be considered here.

Another fundamental difficulty is associated with the decomposition into mean and fluctuations. This essentially assumes that there is a spectral gap of some sort between the mean and fluctuations, allowing an assumption that the mean changes little over a time or space scale large enough for the determination of statistically accurate eddy fluxes. This important issue will be discussed, albeit naively, in Section 2.

A further basic issue concerns the extent to which different ocean processes are in parallel or in series. If in parallel, which is dominant? If in series, which one controls the eddy flux, leaving the other just to do what it has to? This will be discussed in Section 3, followed by a discussion in Section 4 of the possible insights to be gained from a "triple decomposition" into mean, eddies and turbulence. Section 5 addresses the inter-relationship of isopycnal stirring and diapycnal mixing, and Section 6 speculates on the factors that may determine the "mixing efficiency" with which mechanical energy input, such as that from breaking internal waves, is converted to an increased potential energy of the mean state. Some clues on this may come in the future from careful analysis of overturning motions in the ocean, so some recent ideas and results on this topic are summarized in Section 7.

2. The Spectral Gap

At mid-latitudes, internal waves may overlap in spatial scale with other, quasi-geostrophic, motions, but have different time scales. Thus, in determining the effect of the former on the latter, one could presumably exploit the existence of a spectral gap in the frequency domain. Near the equator the internal wave time scales may be longer and overlap with the time scales of other motions, so that a separation does not appear to be possible in either time or space. Nonetheless, a "dynamical gap" still exists, which is what permits one to talk separately about the two classes of motion, and one assumes that it will still be possible to parameterize the effects of the waves on the other motions.

Motions other than internal waves (and even smaller phenomena such as those associated with double diffusive processes) may, of course, have a continuous spectrum, without any spectral gap. This would appear to make parameterization of small scales impossible. The success of Large Eddy Simulations (e.g., *Metais* 1998) has shown, however, that the details of unresolved scales may be unimportant if their only role is to absorb variance that is generated at larger scales and cascades to small scales. This could occur in three-dimensional turbulence, with the only requirement being that the start of the inertial subrange be resolved.

In the ocean, of course, as in the atmosphere, the cascade may be partly to larger, rather than smaller, scales so that the details of the small-scale behavior do affect large scales after a finite time. This is the classic problem of chaotic behavior and lack of predictability. On the other hand, experience in the atmosphere suggests that, while "weather" is unpredictable, "climate" may be predictable (e.g., *Mote and O'Neill* 2000). Maybe we can hope for the same in the ocean.

A naive view might thus be that the lack of a spectral gap is less important than might have been feared. Either separation is possible using a dynamical gap, or unresolved scales do not have a back effect on resolved scales, or the back effect does not affect the climatic state of the ocean. The issue is, however, one that should be kept under review.

3. Are Processes in Parallel or in Series? What is Rate-Controlling?

In three-dimensional turbulence at high Reynolds number, the turbulent scalar flux $\overline{\mathbf{u}C'}$ is in parallel with a much smaller molecular flux $-\kappa\nabla\overline{C}$, with κ the molecular diffusivity. The turbulent flux does depend on the presence of the molecular diffusivity in series; as stressed by *Nakamura* (1996) and *Winters and D'Asaro* (1996), the total flux may be thought of as being purely diffusive, but across a highly convoluted surface of constant concentration. The value of the molecular diffusivity does not determine the value of the turbulent flux; reducing the molecular diffusivity would just lead to an increase in the streakiness, or fine-scale gradients, of the scalar.

Mathematically this can be summarized from the equation for the rate of change of the concentration variance

$$\frac{\partial\overline{C'^2}}{\partial t} + \nabla\cdot\left(\overline{\mathbf{u}}\,\overline{C'^2} + \overline{\mathbf{u}'C'^2} - \kappa\nabla\overline{C'^2}\right) + 2\overline{\mathbf{u}'C'}\cdot\nabla\overline{C} = -2\kappa\overline{\nabla C'\cdot\nabla C'}. \quad (5)$$

This is just the Osborn-Cox formula (*Osborn and Cox* 1972) if C is the temperature. If the time-dependent and divergence terms on the left hand side of (5) are neglected on the grounds of stationarity and spatial ho-

mogeneity, there is a balance between variance production and dissipation. The turbulent eddy flux may remain the same when κ is reduced provided that the mean square gradient of the concentration fluctuations is increased.

One might say that the turbulent and molecular fluxes act in parallel, with the former dominating, but that the processes also act in series, with the turbulent stirring being the rate-controlling process. The molecular diffusion just does what it has to; it is essential but does not control the flux.

One can think of similar situations in the environment. For example, the meridional flux of potential temperature in the atmosphere is controlled by the eddy flux associated with weather systems. The eddy flux relies on the presence of radiative air mass modification in series, but the magnitude of the flux does not depend on the details of this.

On the other hand, the heat flux across the north wall of the Gulf Stream may appear to be just a function of the rate of meander formation and subsequent generation of warm core rings, which then lose their excess heat through air–sea interaction. Many of the rings, however, recirculate and rejoin the Gulf Stream before they have fully decayed (e.g., *Olson* 1991). Presumably, therefore, the northward heat flux in this case depends, at least partly, on the actual strength of the air–sea interaction which acts in series with the initial eddy flux.

4. Triple Decomposition

When there is some small-scale turbulence acting in series with larger scale eddies, some insight may be obtained from a triple decomposition of the concentration field into mean, eddies and turbulence (*Joyce* 1977; *Davis* 1994). It is assumed that there is a spectral gap between the mean and the eddies, and another gap between the eddies and the turbulence. We write $C = C_m + C_e + C_t$, with subscripts m, e and t denoting mean, eddies and turbulence, respectively. For any quantity Q we write $\overline{Q}$ for the average of Q over a time, or space scale, large compared with the scale of the turbulence but short compared with that of the eddies, and $\langle Q \rangle$ for the average of Q over a time or space scale long compared with that of the eddies but short compared with that of the mean state.

Assuming stationarity, so that variances do not change with time, and homogeneity, so that divergence terms may be ignored, it is straightforward to derive

$$\langle \overline{\mathbf{u}_t C_t} \rangle \cdot \nabla C_m + \langle \mathbf{u}_e C_e \rangle \cdot \nabla C_m = -\frac{1}{2} \langle \chi \rangle \qquad (6)$$

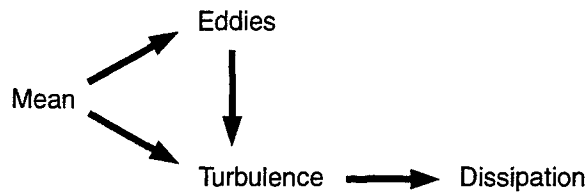

Figure 1. A simple schematic of the path of scalar concentration variance from mean to dissipation via eddies and turbulence.

$$\langle \overline{\mathbf{u}_t C_t} \rangle \cdot \nabla C_m + \langle \overline{\mathbf{u}_t C_t} \cdot \nabla C_e \rangle = -\frac{1}{2} \langle \chi \rangle \qquad (7)$$

$$\langle \mathbf{u}_e C_e \rangle \cdot \nabla C_m - \langle \overline{\mathbf{u}_t C_t} \cdot \nabla C_e \rangle = 0 \qquad (8)$$

where $\chi = 2\kappa \overline{\nabla C' \cdot \nabla C'}$ is the rate of dissipation of scalar variance, as before.

The first equation here is just the Osborn-Cox equation if one does no separate averaging over the time scales, essentially lumping eddy and turbulent fluctuations together. The second equation, however, shows that the dissipation may be regarded as coming from the production of variance by the turbulence acting on both the mean state and the fluctuations produced by the eddies. The third equation is just the difference of the previous two and shows that variance generated by the eddies acting on the mean state must be passed on to dissipation by the turbulence acting on the eddy fluctuations. The situation is summarized in Figure 1, showing the pathway from production to dissipation via the eddies as well as directly from the turbulence acting on the mean state.

The molecular, turbulent and eddy fluxes are all in parallel in determining the total flux at a fixed point, but it is not necessary for the eddy flux to dominate the turbulent flux in the same way as the turbulent flux dominates the molecular flux in the simple case. Just as considered by *Nakamura* (1996) and *Winters and D'Asaro* (1996) for the simple case, one may think of the flux in different ways. At a fixed point, or across the contours of the overall mean scalar, the flux is made up of the molecular, turbulent and eddy fluxes all acting in parallel. Alternatively, the flux is purely molecular across the contours of the instaneous concentration field. A third way of viewing the problem, however, is with respect to contours of C obtained by averaging over the turbulence but not the eddies. In this case the flux is carried by the turbulence and molecular diffusion in parallel. If the contours defined this way are not very convoluted it implies that the transport across a fixed surface is dominated by the turbulence rather than by the eddies.

There are likely to be variations on this theme, but

it does seem that a triple decomposition may provide insight in some situations, bearing in mind, however, that there may not be convenient spectral gaps between the mean, eddies and turbulence.

5. The Relationship Between Isopycnal Stirring and Diapycnal Mixing?

We tend to think of adiabatic processes associated with mesoscale eddies in the ocean as being independent of small-scale diabatic mixing processes caused by things like breaking internal waves. It is important to consider whether this is true, and, if not, what implications it might have for the parameterization of diapycnal mixing. Two previously studied problems will be briefly reviewed here. The first concerns the ultimate fate of the available potential energy released into eddies in baroclinic instability; the second concerns the final dissipation mechanism for thermohaline fronts that are presumably generated by adiabatic stirring on isopycnal surfaces.

5.1 Does baroclinic instability lead to diapycnal mixing?

A popular parameterization scheme for the relaxation of mean isopycnals by baroclinic instability in the ocean was proposed by *Gent and McWilliams* (1990). It represents the horizontal components of the skew velocity by $(\kappa\nabla_h b/b_z)_z$, where b is the buoyancy field. *Gent et al.* (1995) showed that the effect of this is close to that of having a vertical eddy viscosity $A_v = (f^2/N^2)\kappa$ acting on the mean flow. The appropriateness of this scheme over one invoking the mixing of potential vorticity is discussed elsewhere in this volume. The point to be made here is that, as discussed by *Tandon and Garrett* (1996), one needs to consider the ultimate fate of the mean available potential energy released to the eddies. It seems unlikely that it is dissipated adiabatically by internal friction in the ocean. Some of the eddy damping may occur via air–sea interaction or by viscous damping in the boundary layer at the sea floor, but one also needs to consider the possibility that the energy is lost in the fluid interior and that some fraction of this appears, through diapycnal mixing, as an increase of the basic potential energy.

As discussed by *Tandon and Garrett* (1996), the rate of release from the mean flow is $(f^2/N^2)\kappa|\mathbf{u}_{hz}|^2$ for an equivalent eddy viscosity, as above, acting on the vertical shear $\mathbf{u}_{hz}$ of the mean horizontal flow $\mathbf{u}_h$. Using the thermal wind equation $|\mathbf{u}_{hz}| = f^{-1}|\nabla_h b|$ this energy loss rate becomes $s^2\kappa N^2$ where $s = |\nabla_h b|/N^2$ is the mean isopycnal slope. If the rate of creation of mean potential energy is at a rate R_f times this, and is expressed as $K_v N^2$, then the vertical eddy diffusivity is $K_v = R_f s^2 \kappa$ and could be as large as $10^{-3}\mathrm{m}^2\mathrm{s}^{-1}$ in places like the Southern Ocean. *Tandon and Garrett* (1996) argue that this could have a significant effect on tracers, though it is unlikely to significantly augment the spin-down of the mean flow already associated directly with the Gent and McWilliams mechanism.

The problem thus seems to warrant continued investigation. Strong internal wave activity and inferred high dissipation have been reported for the Southern Ocean by *Polzin and Firing* (1997). They suggest that the waves originate as lee waves generated by strong flows over a rough sea floor, and that these strong flows include currents associated with the mesoscale eddies. In that case the internal waves would indeed be associated with the decay of the baroclinic eddies, though the mixing in the water column would not be associated with the local current shear. It is worth considering whether there could additionally be some direct local connection between the mesoscale eddy current shear and the excitation of internal waves.

5.2 Are thermohaline fronts dissipated passively or actively?

Compensating lateral gradients of potential temperature and salinity can exist on isopycnal surfaces in the ocean. Purely adiabatic stirring of these gradients will presumably lead to density-compensated thermohaline fronts. Some small-scale mixing mechanisms must occur to prevent the temperature and salinity gradients from increasing without limit.

The gradients could, of course, become sharp enough that molecular diffusion alone removes them. Alternatively, there might be some small-scale lateral mixing mechanism, perhaps involving vortical modes, which would hasten the transfer to molecular scales. Both of these possibilities could be described as "passive" in the sense that the small-scale mixing mechanism was in place already, and just copes with the extra variance produced by the isopycnal stirring.

Another, more likely, passive mechanism was explored by *Haynes and Anglade* (1997) and is reviewed by Haynes in this volume. The fundamental idea is that the vertical shear of the stirring process leads to the development of vertical scalar gradients at the same time as lateral gradients. Sharp gradients develop in the vertical as well as laterally, with diapycnal mixing then acting to smooth them. *Haynes and Anglade* (1997) argue that a lateral frontal width of a few kilometres is a plausible outcome for reasonable values of the vertical shear.

An alternative mechanism for the thermohaline frontolysis was proposed by *Garrett* (1982) and reviewed

by *Garrett* (1989). He suggested that as the front develops, it will be unstable to thermohaline intrusions which would limit the further narrowing of the front, effectively dissipating the lateral variance production by larger-scale stirring. This might be termed an "active" process in that it introduces extra diapycnal mixing that would not have occurred without the lateral stirring. The associated diapycnal eddy diffusivity is inevitably negative for density (as is the case for density for any process driven double-diffusively), but is likely to be positive for salinity and can take either sign for temperature, depending on the circumstances. The process deserves further study if the effective diapycnal diffusivities that one would add to a model for large-scale oceanic behavior reach significant values. If the diffusivities are very small one would argue that the process is interesting scientifically but just does what it has to without having any large-scale impact.

The formula suggested for the effective diapycnal diffusivity of salt is

$$K_s = 10^{-3} D^2 N (g\beta |\nabla_h S|/N^2)^3 \qquad (9)$$

where D is the diameter of the mesoscale eddies doing the stirring, β the density coefficient for salinity and $|\nabla_h S|$ is the lateral gradient of the large-scale salinity field (over a scale larger than that of the eddies causing the stirring). The value of K_s is clearly very sensitive to the lateral salinity gradient and only likely to be significant in a few places such as the Mediterranean salt tongue where it is large (*Garrett*, 1989).

The model also predicts the thickness of the intrusions to be about $\frac{1}{2}(g\beta|\nabla_h S|/N^2)D$. This could be compared with data in any systematic examination of CTD profiles. Of course the intrusions would only be found in the narrow frontal regions. The width of these was predicted to be

$$W \simeq 0.08(g\beta|\nabla_h S|/N^2) D N \Omega^{-1} \qquad (10)$$

where Ω is the large scale strain rate (say 10^{-6}s^{-1} or less typically).

Presumably this active frontolytic mechanism only occurs if the width from (10) is larger than the width in the passive mechanism proposed by *Haynes and Anglade* (1997). The particular values for W tabulated by *Garrett* (1989) show that this may be the case, though clearly there is considerable uncertainty in both models.

One location for which information is available is the site of the North Atlantic Tracer Release Experiment (*Ledwell et al.* 1993). In this experiment an artificial tracer injected at a depth of about 300 m was teased out into streaks by a strain rate estimated as $3 \times 10^{-7}\text{s}^{-1}$ and reached a width of about 3 km. This width was found by *Haynes and Anglade* (1997) to be plausible for the mechanism they described, though with considerable uncertainty associated with the lack of information on the vertical shear of eddy currents.

To apply the model considered here, we need to assume that at the same time as the tracer became streaky, the associated lateral convergence would lead to thermohaline frontogenesis. The large-scale isopycnal salinity gradient in this region is approximately 10^{-7}m^{-1}, and $N^2 \simeq 1.8 \times 10^{-5}$ so that $g\beta|\nabla_h S|/N^2 \simeq 4 \times 10^{-5}$. If we take $D \simeq 100$ km, then (10) gives a frontal width of about 5 km. This is also close to the observed width. A point against this interpretation is that no intrusive features were reported, though *Garrett's* (1982) model would predict them to be only 2 m in vertical extent with very small temperature and salinity signatures and so perhaps not readily observable.

It also seems quite likely that in this particular situation neither of the models discussed here constituted the ultimate lateral mixing process, but that the mixing was actually accomplished by small-scale vortical modes. The discussion is perhaps somewhat academic anyway; even if *Garrett's* (1982) mechanism were the ultimate frontolytic process in the NATRE region, the regional average diapycnal mixing rate for salt, using (9), would be less than $10^{-8}\text{m}^2\text{s}^{-1}$ and so utterly negligible. (The smallness of this value is partly associated with the small area occupied by the thermohaline fronts.)

Perhaps the realistic conclusion at this stage is that we do not know in general what the frontolytic mechanism is, that it may vary from place to place, and that in a few situations it may be important to know what it is if it introduces further diapycnal mixing.

6. Mixing Efficiency

An important issue in considering the relationship of stirring to mixing in the diapycnal sense is that of the mixing efficiency: how much of the energy being put into stirring motions is converted to the potential energy of the basic state? The topic is discussed by Barry elsewhere in these proceedings and also by *St. Laurent and Garrett* (2001).

The standard approach is to write

$$K_v = \Gamma\epsilon/N^2, \qquad \Gamma = R_f/(1-R_f), \qquad (11)$$

where R_f is the flux Richardson number, representing the fraction of energy put into the stirring motions that produces a diapycnal buoyancy flux rather than being dissipated.

It is customary to choose Γ to be about 0.2, based on comparisons of different methods of measuring the di-

apycnal diffusivity (e.g., *Oakey* 1982), but there seems to be no reason why it should be a universal constant. *Ivey and Imberger* (1991) argue that Γ varies considerably in a two-dimensional space described by two dimensionless parameters. One is the "overturn Froude number" $Fr_T = (L_R/L_C)^{2/3}$, where $L_R = (\epsilon/N^3)^{1/2}$ is the Ozmidov scale and L_C is "the scale of the most energetic overturns". The other parameter is the "overturn Reynolds number" $Re_T = (L_C/L_K)^{4/3}$, where $L_K = (\nu^3/\epsilon)^{1/4}$ is the Kolmogorov scale.

We note, however, that Fr_T is an "internal" dimensionless parameter, with its value determined by the turbulent mixing itself rather than by the physical processes driving the mixing. It thus does not seem to be a satisfactory descriptor. We might also expect that Γ should be independent of Re_T for large values of the latter or, equivalently, of the turbulent Reynolds number $\epsilon/(\nu N^2) = Fr_T^2 Re_T$ when this is large.

Thus one still seeks "external" dimensionless parameters on which Γ might depend. One wonders whether Γ might be less if the shear is persistent (compared with N^{-1}) at a particular location; then for much of the time the turbulence is mixing water that is already well mixed, hence with less buoyancy flux resulting. This persistence might be associated with the frequency content of the internal wave spectrum (with an enhancement of low frequency energy, such as that at inertial and tidal frequencies) increasing the persistence and reducing Γ. Alternatively, persistence could be a consequence of greater exceedance of some stability criterion: if the Richardson number goes well below 1/4 at its minimum, then its duration below 1/4 is increased over a case in which it just barely goes below 1/4.

Perhaps an apparent dependence of Γ on $\epsilon/(\nu N^2)$ might just be because this parameter is acting as a proxy for some other indicator of mixing strength that does not involve the viscosity ν.

A test in the field could of course come from simultaneous data on ϵ and the temperature microstructure dissipation rate χ if both were available, as in *Oakey* (1982). If only velocity microstructure data and coarser density data were available one could proceed via a comparison of Thorpe and Ozmidov scales. The former, L_T, is the r.m.s. displacement in reordering a density profile to be stable. The latter, L_O, is defined as $(\epsilon/N^3)^{1/2}$. If we write $L_O = \alpha L_T$, then $K_v = \Gamma\epsilon/N^2$ implies that

$$K_v = \Gamma\alpha^2 N L_T^2. \tag{12}$$

With $\Gamma \simeq 0.2$ and $\alpha \simeq 0.79$ (*Dillon* 1982) or 0.66 (*Crawford* 1986) this gives $K_v \simeq 0.1 N L_T^2$. This latter result seems likely to be a consequence of the dynamics of overturning and more likely to be a general result than a fixed proportionality between L_O and L_T. Hence if α is found to differ from 0.8, one could use (12) to imply $\Gamma \simeq 0.1\alpha^{-2}$. Interestingly, *Ferron et al.* (1998) found $\alpha \simeq 0.95$ in the energetic abyssal mixing in the Romanche Fracture Zone. As they point out, the error bars on all these values of α are such that they could be consistent with a universal value, but we note that the higher value they find might also imply a lower mixing efficiency.

7. The Probability Distribution of Thorpe Displacements

The existence of statically unstable portions of a water column is clear evidence for vertical stirring and mixing. The nature of this stirring is usually characterized by the single parameter of the Thorpe scale, as discussed above. Recently, however, *Stansfield et al.* (2001) have suggested that more information might be available from the actual probability distribution $P_1(L)$ of the Thorpe displacement L (the particle displacement in the reordering process). The Thorpe scale L_T is simply $\left(\int_0^\infty L^2 P_1(L)dL\right)^{1/2}$. Now

$$P_1(L) = \int_0^\infty H P_3(L/H) P_2(H) dH \Big/ \int_0^\infty H^2 P_2(H) dH \tag{13}$$

where $P_2(H)$ is the probability distribution of overturns of thickness H (an overturn being a closed set of displacements) and $P_3(L/H)$ is the normalised probability distribution of displacement L.

Stansfield et al. (2001) do not offer any model for $P_2(H)$, but suggest a very simple kinematic model for $P_3(L/H)$, based on the assumption that each particle has an equal probability of going to any other location in the overturn. This is equivalent to the assumption that any rearrangement of the elements that make up an overturn is equally likely. If the overturn is made up of n points, then we may take $L/H = m$ for $0 \le m \le n$ and it is easy to show that $P(0) = 1/n$ and otherwise $P(m) = 2(n-m)/n^2$. Then $L_T^2 = H^2/6$ for large n.

There are subtleties associated with this model as it is necessary to exclude the rearrangements that do not constitute a single overturn, but these are not critical if n is larger than about 20 or so. Figure 2 shows $P_3(L/H)$ for winter and summer data from Juan de Fuca Strait. While the error bars are not shown here, there are clearly more small displacements, and fewer medium and large displacements, in the data than in the model. There is little reason to regard the model as anything other than a convenient reference, but it seems possible that examination of $P_3(L/H)$ and also $P_2(H)$ in different settings might reveal different mixing regimes and possibly provide an indicator of different mixing efficiencies.

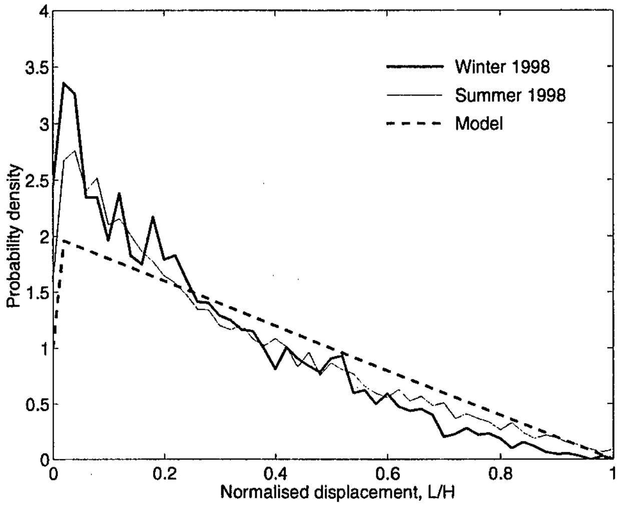

Figure 2. The probability distribution of displacements, normalised by the overturn height, from winter 1998 (thick line) and summer 1998 (thin line) data from Juan de Fuca Strait. The dashed line is the prediction of a simple kinematic model, assuming 50 points in a profile (though this number only affects the value of L/H at which the model prediction rises to from 1 to 2.) From Stansfield et al. (2001).

Conclusions

This short review has attempted to raise some of the issues of stirring and mixing in the ocean, bearing in mind that a primary goal is to derive parameterizations of small-scale processes that have a significant impact on important aspects of the ocean's behavior. An overriding issue is the need for formulae, not just numbers, for any parameterization in a model that aspires to predictive capability. If, in a model of a particular resolution, several sub-grid-scale processes appear to be acting together, it is clearly important to decide which processes are rate-controlling so that theoretical and observational programs can be focused on them.

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